MATH 290 Introduction to Advanced Mathematics SYLLABUS

Prof. Sachs Spring 2018

TEXT: No printed textbook required. Notes from me along with access to an open source book for some initial material will be the "textbook". A link to the online book is on Blackboard. My notes will be forthcoming on Blackboard also.

COURSE OVERVIEW: This course aims to prepare students for later courses in advanced undergraduate mathematics. The goal is therefore "mathematical maturity" which includes clear reasoning, writing and speaking about mathematics, proving results, but also building skills in problem solving, investigation, conjecture, and making connections to prior knowledge. The processes of abstraction and generalization will be explicitly acknowledged during the course. A range of proof techniques will appear and we will discuss the thinking behind these techniques.

The key content of our experimental variant will be the complex numbers and how the complex setting is simultaneously similar to and yet quite different from corresponding real number situations. Building on some high school experiences and calculus of one real variable, we will investigate specific examples and develop general statements about complex polynomials, rational functions, and some of the transcendental functions familiar in calculus for a real input. There will be glimpses of algebra, analysis, geometry, number theory, combinatorics, and topology in our work, along with a dose of history.

WARNING: We are experimenting with this course content. The proposed topics and schedule is definitely subject to change based on your feedback.

MEETING: Tues. and Thurs. Noon – 1:15 pm, Robinson B106

OFFICE HOURS: Tues. and Thurs., 10:45-11:45am, Exploratory 4211, and by appt.

CONTACT INFO: OFFICE PHONE: 703-993-1464 E-MAIL: rsachs@gmu.edu
Use headers to avoid spam filtering!

LA: Ms. Xi Tang, xtang5@masonlive.gmu.edu

COURSE WEB PAGE: Blackboard page at mymasonportal

GRADING: The grading scale is as follows, and is based on your correctly rounded semester average. There will likely be no curve. A+: 98+ A: 93 - 97; A-: 90 - 92; B+: 88 - 89; B: 83 - 87; B-: 80 - 82; C+: 78 - 79; C: 73 - 77; C-: 70 - 72; D: 60 - 69; F: 0 - 59 Grading will be fair and impartial. Points used as the basis of the grade will be: Hmwk. (400 pts.); Class Write-ups (250 pts.); Midterm Exam (150 pts.); Final exam (200 pts.).

Writing Assignments: There will be approximately 10 short writing assignments given throughout the semester. The assignments will involve writing mathematically and grammatically correct solutions to problems, usually involving proofs. Your grade for these assignments will be based on the correctness of your proofs and clarity and correctness of your writing. You will receive feedback on your writing assignments which will total at least 3500 words according to the guidelines of the Writing Across the Curriculum Committee. Precise assignments and due dates will given on BlackBoard. Collaboration is not permitted on these assignments.

Collaborative Work: Most class periods there will be assigned several problems whose solution will take the form of collaborative work done primarily in class. The work will be done in groups of no more than three that either I will assign or that you will choose. Some class periods there will be a new assignment given and in others you will be asked to finish the assignment given out previously. You will be asked to write up the solutions you come up with as a group and submit them. These submissions will be subject to the same requirements as the homework assignments in the previous item and will be graded in the same way. Your score on these assignments will comprise 250 points.

OTHER POLICIES: The GMU Honor code is in effect at all times and students are expected to be fully aware of its requirements. Group work may be part of the course, in which case group members will truthfully report on non-contributing members. Absence from quizzes and exams must be for a valid reason and requires prior notification except in extreme circumstances. DO NOT ARRANGE TO LEAVE BEFORE THE FINAL EXAM. If you are a student with a disability and you need academic accommodations, please see me and contact the Office of Disability Resources at 703/993-2474. All academic accommodations must be arranged through that office.

EXAMS: Midterm Exam **Tentative** Tuesday, March 6
Final Exam **Definitely** Thurs. 5/10 10:30 am 1:15 pm

MATERIAL COVERED AND TENTATIVE WEEKLY SCHEDULE This schedule is likely too rapid so we will adjust as needed.

- Week 1: Overview of course; basics of complex numbers; the complex plane – coordinates, vector view, polar coordinate view. Complex conjugation. Modulus of complex number. Linking properties of complex numbers to real numbers and/or vectors in 2-D. What are the Gaussian integers?
- Week 2: Multiplication and its algebra and geometry continued; complex linear functions; squaring and square roots; quadratic formula revisited; integer powers of z as functions and solving $z^n = w$ for z given w. How roots are related for powers.
- Week 3: Polynomials – definitions; some basic properties; multiplying polynomials; division and the Euclidean algorithm; roots and factors; GCD of two polynomials; division and idea of complex derivative for polynomials.

- Week 4: Cardano's formula for cubics; Bombelli's strategy; solving quartics and the dream of solving all polynomials by radicals. Local view of polynomial functions. Thinking about Fundamental Theorem of Algebra.
- Week 5: Rational functions; the special case of rational functions of degree 1; local view of rational functions.
- Week 6: Euler's formula revisited; the complex exponential function from one viewpoint.
- Week 7: Mid-term Exam; series viewpoint of exponential and the need for derivatives.
- Week 8: Calculus of one variable revisited in the complex plane – basic definitions of limits, continuity, and derivative.
- Week 9: The new meaning of derivatives; geometry of complex differentiable mappings. Some added perspective on Fundamental Theorem of Algebra.
- Week 10: Rational function of degree 1 revisited. Geometric properties. Infinity.
- Week 11: Power series revisited using complex numbers.
- Week 12: Introduction to Fourier series.
- Week 13: The finite Fourier transform.
- Week 14: Some non-Euclidean geometry. Link to special relativity.

Along the way, we will talk about some themes: dimensionality, parametrization, generalization, and use computer software (Mathematica) and on-line applets to aid in visualization and calculation.

Students as Scholars: This class is identified as a Students as Scholars Scholarly Inquiry course. Through the individual written assignments and group work, emphasis will be placed on learning the type of thinking that is involved in understanding advanced mathematical concepts, and in furthering the mathematical enterprise. This includes learning how to formulate conjectures and proof strategies based on evidence gathered from examples, and special cases. In addition, the student will learn the value of the proper formulation of a definition. All of these are basic skills required to understand the motivations and techniques that inform all mathematical research. Information on Students as Scholars is at http://oscar.gmu.edu/ or ask me.